Systems of Equations and Inequalities

8.1 – Linear Systems (Review)
8.6 – Nonlinear Systems

Objectives:

- 8.1: Solve applications of linear systems
- 8.6: Graph systems of nonlinear equations to find points of intersection
- 8.6: Solve systems of nonlinear equations using algebraic methods
- 8.6: Use systems of nonlinear equations to solve applications

System of Linear Equations

A movie theater sells tickets for \$10 each, and student tickets are \$8 if you have your Valencia ID. One evening, the theater sold a total of 525 tickets and took in \$4820 in revenue. How many of each type of ticket was sold?

x - non-student
y - student

$$\begin{cases} -10(x + y = 525) \\ 10x + 8y = 4820 \end{cases}$$

$$\begin{array}{r} \cancel{-10x} - 10y = -5250 \\ + \ \cancel{10x} + 8y = 4820 \\ \hline -2y = -430 \\ y = 215 \end{array}$$

$x + y = 525$

$x + 215 = 525$

$x = 310$

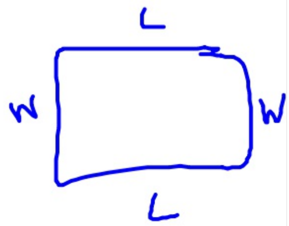
System of Linear Equations

The perimeter of a rectangular floor is 100 feet. Find the dimensions of the floor if the length is four times the width.

L - length
W - width

$$\begin{cases} 2(L) + 2W = 100 \\ L = 4W \end{cases}$$

sub. in for L



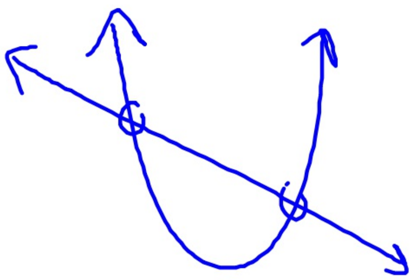
$$2(4W) + 2W = 100$$
$$10W = 100$$
$$W = 10$$

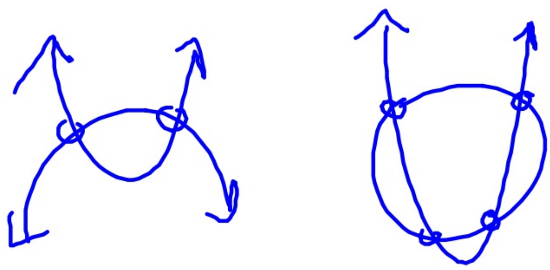
$$L = 4W$$
$$L = 4(10) = 40$$

$$\boxed{10\text{ ft by }40\text{ ft}}$$

Systems of Non-linear Eqautions

The graphs of these systems can include any non-linear function, therefore there can be more than one solution.





Importance of Graphing and Checking Solutions

1. If the system contains two variables and if the equations in the system are easy to graph, then graph them. By graphing each equation in the system, you can get an idea of how many solutions a system has and approximately where they are located.

2. Extraneous solutions can creep in when solving nonlinear systems, so it is imperative that all apparent solutions be checked.

Example 1: Solving a System of Nonlinear Equations Using Substitution

Solve: $\begin{cases} 2x + y = -6 \\ x^2 + 2y = 0 \end{cases}$

Write your answers as ordered pairs separated with commas.

$$2\begin{cases} (2x + y = -6) \\ x^2 + 2y = 0 \end{cases} \rightarrow \begin{array}{r} 4x + \cancel{2y} = -12 \\ -\;\; x^2 + \cancel{2y} = 0 \\ \hline 4x - x^2 = -12 \end{array}$$

$$0 = x^2 - 4x - 12$$

$$0 = (x+2)(x-6)$$

$$x = -2, 6$$

when $x = -2$

$$2x + y = -6$$
$$2(-2) + y = -6$$
$$-4 + y = -6$$
$$y = -2$$

$\boxed{(-2,-2)}$

when $x = 6$

$$2(6) + y = -6$$
$$12 + y = -6$$
$$y = -18$$

$\boxed{(6,-18)}$

Your Turn

Solve the following system by substitution.

Solve: $\begin{cases} x = 3y \\ x = y^2 + y \end{cases}$ sub. in for x

Write your answers as ordered pairs separated with commas.

$3y = y^2 + y$

$0 = y^2 - 2y$

$0 = y(y-2)$

$y = 0, 2$

when $y=0$

$x = 3(0) = 0$ $\boxed{(0,0)}$

when $y=2$

$x = 3(2) = 6$ $\boxed{(6,2)}$

Example 2: Solving a System of Nonlinear Equations Using Elimination

Solve: $\begin{cases} \cancel{x^2} + y^2 = 8 \\ - \cancel{x^2} + 2y = 0 \end{cases}$

Write your answers as ordered pairs separated with commas.

$y^2 - 2y = 8$

$y^2 - 2y - 8 = 0$

$(y+2)(y-4) = 0$

$y = -2, 4$

when $y = -2$

$x^2 + 2(-2) = 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

$\boxed{(2,-2), (-2,-2)}$

when $y = 4$...

$x^2 + 2(4) = 0$

$x^2 + 8 = 0$

$x^2 = -8$ non-real solutions

Your Turn

Solve the following system by elimination.

Solve: $\begin{cases} x^2 - 2y^2 + 14 = 0 \\ 4x^2 + y^2 = 25 \end{cases}$ $\rightarrow$ $\begin{cases} x^2 - 2y^2 = -14 \\ 2(4x^2 + y^2 = 25) \end{cases}$ $\rightarrow$ $\begin{cases} x^2 - 2y^2 = -14 \\ + \; 8x^2 + 2y^2 = 50 \end{cases}$

Write your answers as ordered pairs separated with commas.

$9x^2 = 36$

$x^2 = 4$

$x = \pm 2$

when $x = \pm 2$... $x^2 - 2y^2 = -14$

$(\pm 2)^2 - 2y^2 = -14$

$4 - 2y^2 = -14$

$-2y^2 = -18$

$y^2 = 9$

$y = \pm 3$

$\boxed{(2, 3)\ (-2, 3)\ (2, -3)\ (-2, -3)}$

Example 3: Solving a System of Nonlinear Equations

Solve: $\begin{cases} 2x + y = -6 \rightarrow y = -2x - 6 \\ x^2 + y^2 = 1 \end{cases}$

Sub. in for y

Write your answers as ordered pairs separated with commas.

$$x^2 + (-2x-6)^2 = 1$$

$$x^2 + (-2x-6)(-2x-6) = 1$$

$$x^2 + 4x^2 + 24x + 36 = 1$$

$$5x^2 + 24x + 35 = 0$$

(not factorable! ... use formula)

$$x = \frac{-24 \pm \sqrt{24^2 - 4(5)(35)}}{2(5)}$$

$$= \frac{-24 \pm \sqrt{576 - 700}}{10} \leftarrow \text{non-real!}$$

$\emptyset$

Example 4: Solving a System of Nonlinear Equations

Solve $\begin{cases} x^2 + 4y^2 = 24 \\ 4(6x^2 - y^2 = 10) \end{cases} \rightarrow \begin{cases} x^2 + \cancel{4y^2} = 24 \\ + \; 24x^2 - \cancel{4y^2} = 40 \end{cases}$

Write your answers as ordered pairs separated with commas.

$25x^2 = 64$

$x^2 = \frac{64}{25}$

$x = \frac{\pm 8}{5} = \pm 1.6$

$6x^2 - y^2 = 10$

$6\left(\frac{64}{25}\right) - y^2 = 10$

$\frac{384}{25} - y^2 = 10$

$-y^2 = \frac{250}{25} - \frac{384}{25}$

$+y^2 = \frac{+134}{25}$

$y = \frac{\pm\sqrt{134}}{5}$

solutions

$\left(\frac{8}{5}, \frac{\sqrt{134}}{5}\right)$

$\left(\frac{8}{5}, -\frac{\sqrt{134}}{5}\right)$

$\left(-\frac{8}{5}, \frac{\sqrt{134}}{5}\right)$

$\left(-\frac{8}{5}, -\frac{\sqrt{134}}{5}\right)$

Example 5: Application

The product of two numbers is 64 and the sum of their squares is 128. Find the numbers. Write your answer as an ordered pair with the larger number listed first.

$$\begin{cases} xy = 64 \\ x^2 + y^2 = 128 \end{cases}$$

$$y = \frac{64}{x}$$

$$x^2 + \left(\frac{64}{x}\right)^2 = 128$$

$$x^2\left[x^2 + \frac{64^2}{x^2} = 128\right]$$

$$x^4 + 64^2 = 128x^2$$

$$x^4 - 128x^2 + 64^2 = 0$$

$$(x^2 - 64)(x^2 - 64) = 0$$

$$x^2 - 64 = 0$$

$$x^2 = 64$$

$$x = \pm 8$$

$$y = \frac{64}{8} = 8$$

$$y = \frac{64}{-8} = -8$$

$$\boxed{\begin{matrix}(8, 8) \\ (-8, -8)\end{matrix}}$$